

EXAMINATION PAPER 13+ Academic Scholarship 2023

Mathematics Paper 2

Time allowed: 2 hours

Instructions

- Calculators are NOT allowed.
- You **are** expected to answer all of Questions 1 6.
- You **are not** expected to have time to answer all of Questions 7 14.
 - You may answer these questions in any order.
 - o Choose those questions which you think you can answer best.
 - You may not need all the space provided to answer the question.
- Remember to show your working and clearly show the method you are using in all questions.

Question 1 Work out the following:

a)
$$6.5 \times 4.32$$

c) $\frac{2+\frac{1}{2}}{3-\frac{1}{3}}$

d) $167 \times 34 + 17 \times 66$

Find the missing terms (?) in each of the following sequences:

b) 18, 12, 8, $5\frac{1}{3}$, ?,...

c) 3, 24, 81, ?, 375, ...

d) ?, 1, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{2}{5}$, ?,...

Question 3 Simplify the following expressions fully:

a)
$$4a + 3ab - 4b - a + 2b$$

b) $5x^2 - 2x(4+2x)$

c) $63x^3 \div 9x^2$

d) 6x - 4(2+x) - (2x-3)

Question 4 If $a = \frac{3}{4}$, b = -3, and c = 8, find the value of the following expressions. Leave your answers as simplified mixed numbers where necessary.

a) 2a-3b

b) ac^2

c) $\frac{2c}{ch+2}$

d) $\frac{4(2b+3)^2}{}$

Question 5 Solve the following equations:

a)
$$\frac{3}{2x} - 7 = 0$$

b) $2x - 4 = \frac{x+6}{5} + 2$

c) x(x-3) = 3(9-x)

d) 4(x+2) - 3(5-2x) = 7

Question 6

You are given that $\frac{38 \times 27}{8.5 + 0.05} = 120$.

Without doing lots of lengthy calculations, write down the values of:

a) $\frac{0.38 \times 2.7}{8.5 + 0.05}$

c) 19×135

Robert goes shopping and buys three shirts and a tie, costing a total of £53. After paying he changes his mind, so he returns the tie and buys an extra shirt instead. With the refund included, he only pays £7 in the second transaction.

Work out the individual cost of each shirt and each tie.

Two normal six-sided dice, one coloured red and one coloured blue, have been modified. On the blue die the number 3 has been replaced by a 4, and on red die the number 4 has been replaced by a 3.

Both dice are thrown.

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- a) What is the probability of rolling a 4 on the blue die and a 3 on the red die?
- b) What is the probability that the total on the two dice is an odd number?

A small bakery bakes the same number of white and brown loaves of bread each day. On one day the bakery sells 120 white loaves and 160 brown loaves. At the end of the day the ratio of the unsold loaves was 5:3 (white: brown). How many loaves did they have at the start of the day?

In a Six Nations Rugby match, the Irish team had a pack of 8 players whose mean weight was 102kg. In the second half 4 of these players, whose mean weight was 98kg, were replaced by 4 players whose individual weights were 96kg, 100kg, 108kg, and *X* kg. The mean weight of the pack of 8 players now rose to 104kg.

Find the value of *X*.

It takes John 1 hour and 48 mins to clean the family car, and it takes Jane 1 hour and 12 mins.

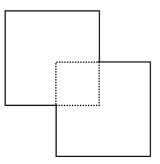
a) If John and Jane work together, how long would it take them to clean the family car?

Richard drives his car 15 miles to the nearest automatic car wash. He drives at an average speed of 50mph.

b) If Richard sets off at the same time that John and Jane start cleaning, and returns just as they finish, how long did the automatic car wash take?

Two identical squares are overlapped to form an irregular octagon, as shown in the diagram below. The overlap also forms a square.

Given that the area of the overlap is $\frac{2}{7}$ of the area of the octagon, what is the ratio of the perimeter of the octagon to the perimeter of one of the original squares?



Question 13 You are given that $2880 = 2^6 \times 3^2 \times 5$

a) Each factor of 2880 can be written using $2^a \times 3^b \times 5^c$ (a, b, and c can be zero; $x^0 = 1$ for all x)

For instance: $10 = 2^1 \times 3^0 \times 5^1$ $12 = 2^2 \times 3^1 \times 5^0$

- i) What would *a* equal if you were writing an odd factor in this way?
- ii) How many values can *b* take?
- iii) Without making a list, how many factors does 2880 have?
- b) Which has more factors, $1,306,800 = 2^4 \times 3^3 \times 5^2 \times 11^2$ or $636,245,792 = 2^5 \times 7^6 \times 13^2$, and how many more?
- c) Write down, in index form, the power of 10 which has exactly 100 factors.

In the addition problem below, each letter stands for a **different** digit (0 is never the first digit of any number).

There are 7 solutions in total, but only one solution which **doesn't** contain the digit 8; find this solution.