



OUNDLE

School

EXAMINATION PAPER
Academic Scholarship 2022

Mathematics – Paper 2

Time allowed: 2 hours

Name: _____

Instructions

- Calculators are **NOT** allowed.
- Please write your answers on lined paper.
- You are not expected to have time to do all the questions.
- You may answer the questions in any order
- Choose those questions which you think you can answer best.
- Remember to show your workings and clearly show the method you are using.
- Give answers to 3 significant figures where necessary.

Question 1

A “mathemagician” is performing a “mind-reading” trick. He picks a random member of the audience and hands them a sealed envelope. He tells the “volunteer” to:

1. choose a number:

He hands over a calculator, just to make sure the volunteer doesn't make a calculation error, and then says:

2. add 29 to your number
3. double the result
4. now subtract 8
5. multiply the result by 10
6. let's add 340 to that number
7. times by 5
8. divide what you've got by 100
9. take away the number you first thought of

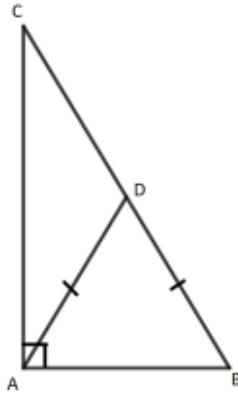
The mathemagician asks the volunteer, “what is the number on the calculator screen?”, to which the response is “42”. The mathemagician tells the volunteer to now open the sealed envelope; the card in the envelope has the number 42 printed in gold lettering; the audience is amazed!

Copy and complete the table below to show how the trick works:

Instruction	Simplified expression
1. choose a number	x
2. add 29 to your number	
3. double the result	
4. now subtract 8	
5. multiply the result by 10	
6. let's add 340 to that number	
7. times by 5	
8. divide what you've got by 100	
9. take away the number you first thought of	42

Question 2

The diagram below shows a right-angled triangle ABC , with $\hat{BAC} = 90^\circ$ (the diagram is not drawn to scale). The point D is added to the side BC so that the line segments $AD = BD$. Show that the triangle ADC is also an isosceles triangle; fully explain your reasoning.



Question 3

You are given that $300 = 2^2 \times 3 \times 5^2$

a) Each factor of 300 can be written using $2^a \times 3^b \times 5^c$ (a, b , and c can be zero; $x^0 = 1$ for all x)

For instance: $10 = 2^1 \times 3^0 \times 5^1$
 $12 = 2^2 \times 3^1 \times 5^0$

- What would a equal if you were writing an odd factor in this way?
 - How many values can c take?
 - Without making a list, how many factors does 300 have?
- b) Which has more factors, $1,306,800 = 2^4 \times 3^3 \times 5^2 \times 11^2$ or $636,245,792 = 2^5 \times 7^6 \times 13^2$, and how many more?
- c) Write down, in index form, the power of 2 which has 32 factors.

Question 4

It takes 5 minutes to fill a bath using only the cold tap, and 7 minutes using just the hot tap.

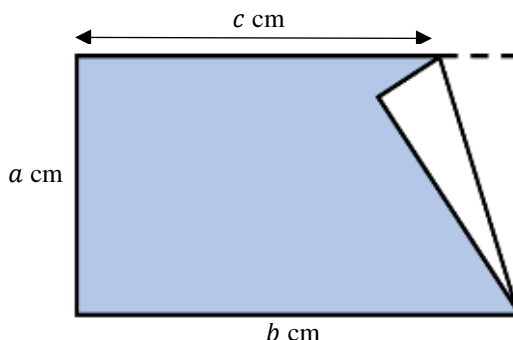
a) How long does it take to fill using both taps at once? Give your answer in minutes and seconds.

A full bath will empty in 3 minutes.

b) How long does it take to fill the bath using both taps at once, but with the plug removed? Give your answer in hours and minutes.

Question 5

A rectangular piece of paper is blue on one side and white on the other. The paper is folded, with the fold starting in the bottom right-hand corner, as shown in the diagram. Show, algebraically, that the ratio $c : b$ is the same as the ratio of the blue area which is still visible : the original area. [You can assume that the folded corner does not go beyond the edge of the paper]



Question 6

At a fish farm, the owner wants to know how many fish there are in one pond. They catch 200 fish and tag each one before putting it back in. The next day they catch 150 fish, 12 of which are tagged. Estimate how many fish are in the pond.

Question 7

A sequence has a first term equal to 6.

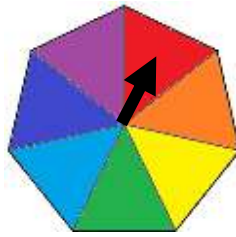
a) Write the next three terms of the sequence if the term-to-term rule is:

- i) add 7
- ii) multiply by 2

b) Instead, the term-to-term rule is “multiply by $\frac{2}{3}$ then add 1”

- i) Write out the first 5 terms, leaving your answers as mixed numbers where appropriate
- ii) Expand and simplify $\frac{2}{3}(3 + x) + 1$
- iii) Using part (ii), or otherwise, explain what happens to the terms in this sequence

Question 8



A game uses the fair spinner above. There is one winning colour, but the game has a bonus round; if you land on a colour next to the winning one then you get to move the pointer one space in either direction.

- What is the probability of winning outright (on the first spin, without using the bonus round)?
- What is the probability of going to the bonus round?
- If you make it to the bonus round, what are your chances of winning?
- What is the overall probability of winning the game?

The maker of the game decides to change the number of sides so that the **overall** chance of winning is 10%.

- How many sides would the spinner need to have to make this happen?

Question 9

In the addition problems below, each letter stands for a **different** digit (0 is never the first digit of any number).

a) Find the solution to each of the following:

(i)	(ii)	(iii)
$\begin{array}{r} A \\ + A \\ \hline B A \end{array}$	$\begin{array}{r} A B \\ + C B \\ \hline B B A \end{array}$	$\begin{array}{r} A B C \\ + C B C \\ \hline C D E B \end{array}$

b) There are multiple solutions to the following

$$\begin{array}{r} O N E \\ + O N E \\ \hline T W O \end{array}$$

Find the only solution which **doesn't** contain the digits 1 or 2.

Question 10 *In this question you are not expected to complete any large calculations*

A Mersenne Prime is a prime number which can be written as $2^n - 1$ for some integer n

a) One of the numbers below is a Mersenne Prime; deduce which one, giving a clear explanation of your reasoning:

524287, 524288, 524289

A narcissistic number is a number which is equal to the sum of its digits raised to the power of the number of digits.

For example $153 = 1^3 + 5^3 + 3^3 = 1 + 125 + 27$
 $1634 = 1^4 + 6^4 + 3^4 + 4^4 = 1 + 1296 + 81 + 256$

b)

i) Copy and complete the grids below by writing odd or even:

	\times	Odd	Even		$+$	Odd	Even
Odd				Odd			
Even				Even			

ii) One of the numbers below is a narcissistic number; deduce which one, giving a clear explanation of your reasoning:

448834, 548834, 648834

Question 11

Using each of the digits from 1 to 9 exactly once, fill in a copy of this grid so that each row, column, and diagonal has the same sum.

Q11 BONUS – no additional marks will be awarded, so make sure that you have thoroughly checked your answers to all the questions before attempting

If you ignore reflections and rotations, then your solution to Q11 is unique. Explain why this is the case.