

## ACADEMIC SCHOLARSHIP 2021

## MATHEMATICS

## PAPER 2

## Time allowed: 2 hours

## CALCULATORS ARE NOT ALLOWED

## Instructions to candidates

- Do not expect to be able to answer every question.
- Marks available for each question are shown in square brackets at the end of each question.
- More marks will be awarded for full answers to a question rather than several fragments.
- Where appropriate aim to explain your answers as fully as possible.

1. This question is about recurring decimals. For example the decimal $0 . \dot{1}=0.111111 \ldots . .$. corresponds to the fraction $\frac{1}{9}$
(i) What is $\frac{4}{9}$ expressed as a decimal?
(ii) Use your answer to part (i) to express $\frac{2}{45}$ as a decimal.
(iii) What fraction does the decimal $0.5 \dot{4}=0.5444444444 . . . .$. correspond to?
2. i) Find the whole number $a$ such that $\frac{1}{a}+\frac{1}{a}=\frac{1}{11}$
ii) Find two different whole numbers $b$ and $c$ such that $\frac{1}{b}+\frac{1}{c}=\frac{1}{11}$.
3. This question refers to the diagram on the right. Angle $A \hat{E} B=128^{\circ}, C \hat{A} E=E \hat{A} B=x^{\circ}, D \hat{B} C=2 x$ and $B \hat{C} D=5 x$ as shown.
a) Express in terms of $x$ the angle $A \hat{B} E$

b) By considering the angles in the triangle $A B C$, form an equation and find $x$.
[6]
4. 

Express the area which is shaded in the diagram below as a fraction of the area of the whole rectangle. Measurements have been made of the length of the rectangle but not the width.

5. Look at the sequences in the table below

|  | $\mathbf{1}^{\text {st }}$ term | $\mathbf{2}^{\text {nd }}$ term | $\mathbf{3}^{\text {rd }}$ term | $\mathbf{4}^{\text {th }}$ term | $\mathbf{5}^{\text {th }}$ term |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{n}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ |
| Sequence $\boldsymbol{A}$ | 3 | 6 | 9 | 12 | 15 |
| Sequence $\boldsymbol{B}$ | 2 | 8 | 18 | 32 | 50 |
| Sequence $\boldsymbol{C}$ | -1 | 2 | 9 | 20 | 35 |

(i) Find the $6^{\text {th }}$ term and the $7^{\text {th }}$ terms of each sequence.
(ii) Explain in words how Sequence $C$ can be obtained from Sequence $A$ and Sequence $B$.
(iii) Find an expression involving $n$ for the $n^{\text {th }}$ term of Sequence $B$.
(iv) Find the $100^{\text {th }}$ term of Sequence $C$.
6. Five beads each numbered $1,2,3,4$ and 5 respectively are placed in a box marked $X$. Three beads each numbered 1, 2 and 3 respectively are placed in a box marked $Y$.

One bead is taken randomly from box $X$ and then one from box $Y$. The results are used to produce the co-ordinates of a point on the positive $x$-axis and $y$-axis respectively. For example, the point $(2,1)$ would be the result of taking a 2 from box $X$ and a 1 from box $Y$.

In the following questions, leave your answer as a fraction in its simplest form.
Calculate the probability that after one selection from each box point,
(i) the selected point is $(2,1)$
(ii) the selected point lies on the line with equation $x=2$
(iii) the selected point lies on the line with equation $y=3$
(iv) the selected point lies on the line with equation $y=x$
(v) the selected point lies on the line with equation $y=2 x-1$
7. A lake contains 300 fish, of which $c$ are carp and all the others are pike. If 100 more carp are added to the lake, the probability of catching a carp is doubled. How many carp were originally in the lake?
8. Pythagoras's Theorem states that in a right-angled triangle, the sum of the squares of the two shorter sides equals the square of the hypotenuse (the longest side). For the diagram, this can be stated as $a^{2}+b^{2}=c^{2}$. Another way to express this is
Area of square $\boldsymbol{A}+$ Area of square $B=$ Area of square $C$.

Find a similar relationship between the areas of all the squares in the following diagrams. Explain your answer.

9. (a) $n$ is a positive even number. The sum of $n$ and the next two even numbers above it is 54 .
(i) Write down an equation for $n$
(ii) Find $n$.
(b) $\quad m$ is another positive even number. When the even number below $m$ is multiplied with the even number above $m$ the result is 396 .
(i) Write down an equation for $m$.
(ii) Find $m$.
10. (i) A person travels 60 miles by train at 120 miles per hour and then takes a bus for 45 miles at 30 miles per hour. What is their average speed for the journey?
(ii) Three Runners, named Bolt, Colt and Dolt, run a 100 m race. Each travels at their own constant speed throughout the race. They finish the race in alphabetical order. Ten metres separates Bolt from Colt when Bolt crosses the finish line and Ten metres separates Colt from Dolt when Colt crosses the finish line. What is the distance between Bolt and Dolt when Bolt crosses the finish line?

Hints: The answer is not 20 m . speed $=\frac{\text { distance }}{\text { time }}$
11. In the isosceles triangle shown, $C A=C B$.

From $A$, a line $A P$ has been drawn to meet the opposite side $B C$ at right angles.


Show that angle $a$ is always half of angle $c$.
12. A man runs at constant speed once anticlockwise around a track starting at a point S . His coach stands at a fixed point F. A rough graph is drawn to show the shortest distance between the two men against time. An example is shown below.



Draw the corresponding graphs for the following track lay outs and starting positions.
(i)

(ii)


13 The picture shows an equilateral triangle which has had the triangle in the top half shaded (not drawn to scale).


Find a way to divide the remaining unshaded shape exactly into four areas of the same shape and size? A sketch of your answer is sufficient rather than anything exact.

14 In the diagram all of the shapes outlined are squares. Determine what fraction of the area of the large square is red?


15 Use divisibility rules to find the missing number (represented by an asterisk in the answer) to these calculations. In each case make it clear which rules you have used and how you have applied them. No marks will be awarded for calculating the multiplication.
(i) $5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13 \times 14=3632 * 28800$
(ii) $7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16=2905 * 430400$

